

Calculus BC

more Taylor Practice

1. Let f be a function that has derivatives of all orders for all real numbers. Assume that $f(3) = 1$, $f'(3) = 4$, $f''(3) = 6$, and $f'''(3) = 12$.

- Write the third order Taylor polynomial for f at $x = 3$ and use it to approximate $f(3.2)$.
- Write the second order Taylor polynomial for f' at $x = 3$ and use it to approximate $f'(2.7)$.
- Does the linearization of f underestimate or overestimate the values of $f(x)$ near $x = 3$? Justify your answer.

2. Let

$$P_4(x) = 7 - 3(x-4) + 5(x-4)^2 - 2(x-4)^3 + 6(x-4)^4$$

be the Taylor polynomial of order 4 for the function f at $x = 4$. Assume f has derivatives of all orders for all real numbers.

- Find $f(4)$ and $f'''(4)$.
- Write the second order Taylor polynomial for f' at $x = 4$ and use it to approximate $f'(4.3)$.
- Write the fourth order Taylor polynomial for $g(x) = \int_4^x f(t) dt$ at $x = 4$.
- Can the exact value of $f(3)$ be determined from the information given? Justify your answer.

3. (a) Write the first three nonzero terms and the general term of the Taylor series generated by $f(x) = 5 \sin(x/2)$ at $x = 0$.

- (b) What is the interval of convergence for the series found in (a)? Show your method.

(c) **Writing to Learn** What is the minimum number of terms of the series in (a) needed to approximate $f(x)$ on the interval $(-2, 2)$ with an error not exceeding 0.1 in magnitude? Show your method.

4. Let $f(x) = 1/(1-2x)$.

- (a) Write the first four terms and the general term of the Taylor series generated by $f(x)$ at $x = 0$.

- (b) What is the interval of convergence for the series found in part (a)? Show your method.

- (c) Find $f(-1/4)$. How many terms of the series are adequate for approximating $f(-1/4)$ with an error not exceeding one percent in magnitude? Justify your answer.

5. Let $f(x) = \sum_{n=1}^{\infty} \frac{x^n n^n}{n!}$

for all x for which the series converges.

- Find the radius of convergence of this series.
- Use the first three terms of this series to approximate $f(-1/3)$.
- Estimate the error involved in the approximation in part (b). Justify your answer.

6. Let $f(x) = 1/(x-2)$.

- (a) Write the first four terms and the general term of the Taylor series generated by $f(x)$ at $x = 3$.

- (b) Use the result from part (a) to find the first four terms and the general term of the series generated by $\ln|x-2|$ at $x = 3$.

In Exercises 7-20 find a power series to represent the given function and identify its interval of convergence. When writing the power series, include a formula for the n th term.

7. $\frac{1}{1+3x}$

10. $\frac{x}{1-2x}$

8. $\frac{3}{1-x^3}$

11. $\frac{1}{1+(x-4)}$

9. $\frac{1}{4x} = \frac{1}{4} \left(\frac{1}{1+(x-1)} \right)$

12. $\frac{1}{2-x}$ (Hint: Rewrite $2-x$)

13. $f(x) = \frac{1}{1-3x}$

14. $f(x) = \frac{1}{3-x}$

15. $f(x) = \frac{1}{1+x^9}$

16. $f(x) = \frac{1}{1+3x^7}$

17. $f(x) = \frac{1}{1+3x}$

18. $f(x) = \frac{1}{4+3x}$

19. $f(x) = \frac{1}{5-x^2}$

20. $f(x) = \frac{1}{4-2x^3}$